International Journal of Theoretical Physics, Vol. 29, No. 8, 1990

De Sitter Gauge Theories of Gravity

Shao Changgui,[1,2] Wang Chunhou,[3] and H. Dehnen[2]

Received January 25, 1990

The two types of de Sitter gravities are constructed with the fiber bundle technique and some special cases are discussed. Relations among de Sitter, Poincaré, and Lorentz gravity are discussed and the contraction from the de Sitter bundle to the Poincaré bundle is demonstrated. Two types of gravitational gauge field equations are obtained by using the de Sitter-Poincaré and de Sitter-Lorentz actions. The de Sitter effect occurring in the field equations is discussed.

1. INTRODUCTION

Denote the de Sitter (dS) universe (with constant curvature) by the dS sphere S^4_λ. According to whether the dS curvature $\lambda > 0$, <0, or $=0$, S^4_λ is written as S^{4+}_λ, S^{4-}_λ, or S^4_0. The dS spheres S^{4+}_λ, S^{4-}_λ, and S^4_0 are isomorphic to cosets $SO(4,1)/SO(3,1)$, $SO(3,2)/SO(3,1)$, and $ISO(3,1)/SO(3,1)$, respectively. It is apparent that S^4_0 is Minkowski space. Let us denote it by M'. One may consider $SO(4,1)/SO(3,1)$ or $SO(3,2)/SO(3,1)$ as the sphere-pole orbit with four-dimensional projective homogeneous coordinates $\{\xi^a\}=\{\xi^i, \xi^5\}$ $(i=0,1,2,3)$. Let the center of the sphere be the center of projection; then S^{4+}_λ, and S^{4-}_λ, regarded as the dS sphere $\eta_{ab}\xi^a\xi^b=-1/\lambda$, may be embedded into five-dimensional pseudo-Euclidean space $E^5_{(4,1)}$ and $E^5_{(3,2)}$, respectively. Here $a, b, \ldots = 0,1,2,3,5$; $\eta_{ab} = \mathrm{diag}(1,-1,-1,-1,-I)$; $I=-\lambda/|\lambda|$.

De Sitter groups $SO(4,1)$ and $SO(3,2)$ are the transformation groups of dS spheres S^{4+}_λ and S^{4-}_λ. Thus, S^{4+}_λ and S^{4-}_λ have global dS invariance. Set $SO(4,1)\equiv dS(4,1)$, $SO(3,2)\equiv dS(3,2)$; then both of them may be written as $dS(5)$ symbolically. When extending the flat space-time manifold M' with global Poincaré invariance to one with the local Poincaré invariance, we may extend it to Riemann-Cartan (RC) space-time M with local Poincaré

[1]Physics Department, Hubei University, Wuhan, China.
[2]Physics Department, Konstanz University, Konstanz, West Germany.
[3]Physics Department, Huazhong Normal University, Wuhan, China.

0020-7748/90/0800-0885$06.00/0

invariance and build Poincaré gravity (PG) (Shao Changgui and Xu Banqing, 1986). But since the Poincaré group $ISO(3,1)$ is the contraction of the dS group $dS(5)$, we may extend the space-time manifold M to the supersurface $M_{\lambda(x)}$ with local dS invariance by means of the extension of the gauge group $ISO(3,1) \to dS_\lambda(5)$, and then build dS gravity (dSG). Of course, it is usual that we localize the constant-curvature space-time manifold S_λ^4 with global dS invariance as the supersurface $M_{\lambda(x)}$ with local dS invariance, and then build dSG. The contraction (or extension) of the gauge group shall give rise to the contraction (or extension) of the bundle, and then the dSG is different from the PG.

When localizing S_λ^4 as $M_{\lambda(x)}$, the five-dimensional pseudo-Euclidean space E_λ^5 embedding S_λ^4 is localized as the RC space $H_{\lambda(x)}^5$, i.e.,

$$E_\lambda^5 \to H_{\lambda(x)}^5, \qquad S_\lambda^4 \to M_{\lambda(x)}, \qquad H_{\lambda(x)}^5 \supset M_{\lambda(x)}$$

Therefore $\forall x \in M_{\lambda(x)}$, there is a dS sphere $S_{\lambda(x)}^4$ ($\subset E_{\lambda|x}^5$) tangential to $M_{\lambda(x)}$ and there is a tangent Minkowski space M'_x. The space $E_{\lambda|x}^5$ is the tangent space of $H_{\lambda(x)}^5$ at point x and then we may assume $M_{\lambda(x)}$ as an umbilical point supersurface.

The local Lorentz and Poincaré transformations may be realized in M'_x; the local dS transformations may be realized in $S_{\lambda(x)}^4$. According to whether the gauge transformations are realized in terms of the local moving frame or the vector, we may build the Lorentz frame bundle (Shao Changgui *et al.*, unpublished) $L(M) = P(M, SO(3,1))$ or its associated bundle $E_L = E_L(M, M', SO(3,1), L)$, the Poincaré affine frame bundle $P(M) = P(M, ISO(3,1))$ or its associated bundle $E_P = E_P(M, M', ISO(3,1), P)$, and the de Sitter frame bundle $dS(M_{\lambda(x)}) = P(M_{\lambda(x)}, dS_\lambda(5))$ or its associated vector endpoint (on the dS sphere) bundle $E_{dS} = E_{dS}(M_{\lambda(x)}, S_\lambda^4, dS_\lambda(5), dS)$.

In this article the dS frame bundle is discussed in two cases, the de Sitter–Lorentz (dSL) bundle and the de Sitter–Poincaré (dSP) bundle.

2. DE SITTER–LORENTZ GRAVITY DESCRIBED BY FIBER BUNDLE

Let $M_{\lambda(x)}$ be covered by an open covering; after introducing the natural frame ∂_μ, $\forall x \in M_{\lambda(x)}$, its coordinates are x^μ, $\mu = 0, 1, 2, 3$. Choose a dSL frame in the projection center of the dS sphere $S_{\lambda(x)}^4 \subset E_{\lambda|x}^5$:

$$\{e'_a\} = \{e'_i, e'_5\} \tag{1}$$

and its dual frame

$$\{\theta'^a\} = \{e'^i, e'^5\}$$

Then we have the inner products

$$\langle e'_i, e'_j \rangle = \eta_{ij} = \mathrm{diag}(1, -1, -1, -1)$$
$$\langle e'_5, e'_5 \rangle = -I$$
$$\langle e'_i, e'_5 \rangle = 0$$
$$\langle e'_a, e'_b \rangle = \eta_{ab}$$

and we have

$$\theta'^i(e'_j) = \delta^i_j, \qquad \theta'^i(e'_5) = 0, \qquad \theta'^5(e'_i) = 0, \qquad \theta'^5(e'_5) = 1$$

Under this frame the dS sphere $S^4_{\lambda(x)}$ satisfies the condition of local supersphere, i.e.,

$$\eta_{ab}\xi'^a\xi'^b = -1/\lambda, \qquad g_{\mu\nu} = \eta_{ab} W'^a_\mu W'^b_\nu, \qquad \eta_{ab}\xi'^a W'^b_\mu = 0$$

where the points $\{\xi'^a\} \in S^4_{\lambda(x)}$ and $\{W'^a_\mu\} = \{V^i_\mu, 0\}$ are dSL frame coefficients, and V^i_μ are Lorentz vierbein fields. Here $i, j, \ldots$ are the indices of the Lorentz moving frame, and we also have

$$\theta'^i = V^i_\mu \, dx^\mu, \qquad e'_i = V^\mu_i \, \partial_\mu, \qquad V^i_\mu V^\mu_j = \delta^i_j, \qquad V^i_\mu V^\nu_i = \delta^\nu_\mu$$

where $dx^\mu(\partial_\nu) = \delta^\mu_\nu$, $\theta'^i(e'_j) = \delta^i_j$, $\theta'^i(\partial_\mu) = V^i_\mu$, $dx^\mu(e'_i) = V^\mu_i$. Because the group $dS_\lambda(5)$ keeps the bilinear metric of E^5_λ invariant, one has

$$\eta_{ab} = \eta_{cd} A^c_a A^d_b, \qquad A^b_a \in dS_\lambda(5)$$

In view of the local dS invariance of $M_{\lambda(x)}$, the local action of the gauge group $dS_\lambda(5)$ may be realized by the local action (right action) on the dSL frame. Thus, we obtain the transformed frame

$$e_a = A^b_a e'_b, \qquad \theta^a = (A^{-1})^a_b \theta'^b, \qquad A^b_a \in dS_\lambda(5) \tag{2}$$

Here $\theta^a(e_b) = \delta^a_b$, $\langle e_a, e_b \rangle = \eta_{ab}$, and $\theta^a = W^a_\mu \, dx^\mu$, $\partial_\mu = W^a_\mu e_a$, where $W^a_\mu = \theta^a(\partial_\mu)$ is the component of θ^a under ∂_μ.

Let the set $\{e_a\}_{|x}$ of all dSL frames at point $x \in M_{\lambda(x)}$ under the gauge group $dS(5)$ be denoted by $dSL_x(M_{\lambda(x)})$ and we know that $dSL_x(M_{\lambda(x)})$ is isomorphic to the group $ds_\lambda(5)$, i.e., $dSL_\lambda(M_{\lambda(x)}) \sim dS(5)$. Taking the union $dSL(M_{\lambda(x)})$ of $dSL_x(M_{\lambda(x)})$ for all x, i.e.,

$$dSL(M_{\lambda(x)}) = \bigcup_{x \in M_{\lambda(x)}} dSL_x(M_{\lambda(x)})$$

we obtain a dSL frame bundle (Kobayachi and Nomizu, 1963) $dSL(M_{\lambda(x)}) = P(M_{\lambda(x)}, dS_\lambda(5))$, of which $M_{\lambda(x)}$ is the base space and $dS_\lambda(5)$ is the structure group. The frame $\{e_a\}$ in expression (2) may be called a dSL frame; $dSL_x(M_{\lambda(x)})$ is a fiber over point x. One has that $\forall x \in M_{\lambda(x)}$ the bundle projection Π maps the set of frames on the fiber $dSL_x(M_{\lambda(x)})$ onto the point

x. The distribution of the dSL frame $\{e_{a(x)}\}$ on $M_{\lambda(x)}$ is a cross section $\sigma(x)$ on the bundle; in the $dSL(M_{\lambda(x)})$ it gives a submanifold diffeomorphic to $M_{\lambda(x)}$. It is easy to know that

$$\dim dSL(M_{\lambda(x)}) = \dim M_{\lambda(x)} + \dim dS_\lambda(5) = 4 + 10 = 14$$

Now, $\forall u' \in dSL(M_{\lambda(x)})$, u' may be denoted by (x, A), $A \in dS_\lambda(5)$. The right action on $dSL(M_{\lambda(x)})$ under the group $dS_\lambda(5)$ is defined as

$$u = u'B, \qquad B \in dS_\lambda(5)$$

$$u = (x, AB), \qquad A, B, AB \in dS_\lambda(5)$$

Thus, suppose $\{I_a\}$ is a set of natural basis in pseudo-Euclidean space E_λ^5; the u' may be considered as the linear mapping $E_\lambda^5 \to E_{\lambda|x}^5$, and for all $a = 0, 1, 2, 3, 5$, one has $u'I_a = e_a$. The right action on $dSL(M_{\lambda(x)})$ under group $dS_\lambda(5)$ may be realized as

$$E_\lambda^5 \xrightarrow{B} E_\lambda^5 \xrightarrow{u'} E_{\lambda|x}^5$$

Here B is a linear transformation in E_λ^5:

$$I_a \xrightarrow{B} B_a^b I_b$$

Now, $\forall x \in M_{\lambda(x)}$, the superspherical condition under the dSL frame on bundle $dSL(M_{\lambda(x)})$ is

$$\eta_{ab}\xi^a\xi^b = -1/\lambda, \qquad g_{\mu\nu} = \eta_{ab} W_\mu^a W_\nu^b, \qquad \eta_{ab}\xi^a W_\mu^b = 0$$

Here the point $\{\xi^a\} \in S_{\lambda(x)}^4$.

We give a definition of the absolute covariant derivative:

$$DW_\mu^a = W_{\mu /\!/ \nu}^a \, dx^4 = (W_{\mu,\nu}^a - \Gamma_{\mu\nu}^\lambda W_\lambda^a + \mathscr{B}_{\nu b}^a W_\mu^b)\, dx^\nu \tag{3}$$

Here $/\!/$ is the twofold covariant derivative with respect to the natural frame on $M_{\lambda(x)}$ and the dSL frame on the bundle, $\Gamma_{\mu\nu}^\lambda$ is the connection under the natural frame, and $\mathscr{B}_{\mu b}^a$ is the dSL connection on bundle $dSL(M_{\lambda(x)})$. The above dSL connection is equivalent to the following connection defined with the dSL frame:

$$De_a = \mathscr{B}_{\mu a}^b \, dx^\mu \otimes e_b = \mathscr{B}_{ca}^b \theta^c \otimes e_b \tag{4}$$

In order to leave the length of the vector invariant under translations, we demand $d\eta_{ab} = 0$, and we have $\Omega_{ab} = -\Omega_{ba}$, $\Omega_a^b = \Omega_{ac}\eta^{cb}$. Here for the form Ω_a^b with respect to the dSL connection we have

$$\Omega_a^b = \mathscr{B}_{\mu a}^b \, dx^\mu = \mathscr{B}_{ca}^b \theta^c, \qquad \Omega_a^b \in dS(5)$$

On $dSL(M_{\lambda(x)})$ the transformation law of the dSL connection may be obtained through (3):

$$\Omega_a^b \to \tilde{\Omega}_a^b = (A^{-1})_c^b \Omega_d^c A_a^d + (a^{-1})_c^b \, dA_a^c, \qquad A \in dS_\lambda(5)$$

Taking the dSL frame $\{e'_a\} = \{e'_i, e'_5\}$, the connection defined in (4) may be written as

$$\begin{aligned} De'_i &= B'^{j}_{\mu_i} \, dx^\mu \otimes e'_j + \mathscr{B}'^{5}_{\mu_i} \, dx^\mu \otimes e'_5 \\ De'_5 &= \mathscr{B}'^{i}_{\mu_5} \, dx^\mu \otimes e'_i \end{aligned} \tag{5}$$

With respect to this frame, one has the Gauss formula and Weingarten formula:

$$\begin{aligned} De'_i &= \tilde{D}e'_i I\omega_{ij}\theta'^j \otimes e'_5 = B^j_{\mu_i} \, dx^\mu \otimes e'_j - I\omega_{ij}\theta'^j \otimes e'_5 \\ De'_5 &= -\omega_{kj}\eta^{ik}\theta'^j \otimes e'_i \end{aligned} \tag{6}$$

Here ω_{ij} is the second fundamental metric of $M_{\lambda(x)}$:

$$\text{II} = \omega_{\mu\nu} \, dx^\mu \otimes dx^\nu = \omega_{ij}\theta'^i \otimes \theta'^j$$

Since $M_{\lambda(x)}$ is an umbilical point supersurface, then we have $\omega_{\mu\nu} = -|\lambda|^{1/2} g_{\mu\nu}$, $\omega_{ij} = -|\lambda|^{1/2}\eta_{ij}$. Thus, expressions (6) may be written as

$$\begin{aligned} De'_i &= B^j_{\mu_i} \, dx^\mu \otimes e'_j + I|\lambda|^{1/2}\theta'_i \otimes e'_5 \\ De'_5 &= |\lambda|^{1/2}\theta'^i \otimes e'_i = |\lambda|^{1/2} V^i_\mu \, dx^\mu \otimes e'_i \end{aligned} \tag{7}$$

Comparing expression (7) with (5), we obtain

$$\Omega'^j_i = B^j_{\mu_i} \, dx^\mu, \qquad \Omega'^i_5 = |\lambda|^{1/2}\theta^i, \qquad \Omega'^5_i = I\Omega'_{i5}$$

or

$$\mathscr{B}'^j_{\mu_i} = B^j_{\mu_i}, \qquad \mathscr{B}'^i_{\mu 5} = |\lambda|^{1/2} V^i_\mu, \qquad \mathscr{B}'^5_{\mu i} = I\mathscr{B}'_{\mu i 5}$$

Thus, the dSL connection under the cross section $\sigma(x)$ on the bundle $dSL(M_{\lambda(x)})$ is

$$\mathscr{B}'_\mu = (\mathscr{B}'^b_{\mu a}) = \begin{pmatrix} B^j_{\mu_i} & |\lambda|^{1/2} V^j_\mu \\ I|\lambda|^{1/2} V_{\mu_i} & 0 \end{pmatrix} \in ds_\lambda(5)$$

Here $B^j_{\mu_i}$ is the Lorentz connection evaluated on the Lie algebra $so(3, 1)$. The relation between the three fundamental forms

$$\text{I} = g_{\mu\nu} \, dx^\mu \otimes dx^\nu = \langle d\bar{R}, d\bar{R} \rangle$$

$$\text{II} = \omega_{\mu\nu} \, dx^\mu \otimes dx^\nu = -\langle d\bar{R}, de'_5 \rangle$$

$$\text{III} = k_{\mu\nu} \, dx^\mu \otimes dx^\nu = \langle de'_5, de'_5 \rangle$$

is

$$\text{III} = -|\lambda|^{1/2}\text{II} = |\lambda| = \text{I}$$

where $\bar{R}$ is the vector radius of the supersurface $M_{\lambda(x)}$ under the dSL frame, and this relation is unchanged under the transformations of frames on the bundle dSL and arbitrary coordinate transformations on $M_{\lambda(x)}$.

In the tangent space $T_p(dSL(M_{\lambda(x)}))$ at $p=(x, A)$, $A \in dS_\lambda(5)$, we let

$$Z_\mu = \partial_\mu - \tfrac{1}{2}\mathscr{B}_\mu^{ab} X_{ab} \tag{8}$$

be a horizontal lifting basis expanding the horizontal subspace H_p of $T_p(dSL(M_{\lambda(x)}))$. In expression (8) the generators [of group $dS_\lambda(5)$] $X_{ab} = \xi_a\partial_b - \xi_b\partial_a$ satisfy $X_{ab} = -X_{ba}$. Apparently $\dim H_p = \dim M_{\lambda(x)} = 4$, and at the same time the right invariance of Z_μ is required, i.e., $R_B Z_\mu(P) = Z_\mu(PB)$, $B \in dS_\lambda(5)$. We also have

$$\Pi Z_\mu = \partial_\mu, \qquad \Pi[Z_\mu, Z_\nu] = [\partial_\mu, \partial_\nu] = 0 \tag{9}$$

Let $T_p(dSL(M_{\lambda(x)})) = H_p \oplus V_p$; then V_p, the vertical subspace of $T_p(dSL(M_{\lambda(x)}))$, is the tangent space of the fiber $\Pi^{-1}(x)$, and $\dim V_p = \dim dS_\lambda(5) = 10$. Thus, $\{Z_\mu, X_{ab}\}$ may be assumed to be a set of basis in $T_p(dSL(M_{\lambda(x)}))$.

Their commutation relations are

$$\begin{aligned} &[X_{ab}, X_{cd}]_f^e = \tfrac{1}{2} f_{ab,cd}^{hk} (X_{hk})_f^e \\ &[Z_\mu, Z_\nu] = -\mathscr{F}_{\mu\nu}^\lambda Z_\lambda - \tfrac{1}{2}\mathscr{F}_{\mu\nu}^{ab} X_{ab} \\ &[Z_\mu, X_{ab}] = 0 \end{aligned} \tag{10}$$

Here

$$f_{ab,cd}^{ef} = \eta_{ad}\delta_b^e\delta_c^f - \eta_{bd}\delta_a^e\delta_c^f + \eta_{bc}\delta_a^e\delta_d^f - \eta_{ac}\delta_b^e\delta_d^f$$

are structure constants of the group $dS_\lambda(5)$. From the second expression of (9), we obtain

$$\Pi[Z_\mu, Z_\nu] = \Pi(h[Z_\mu, Z_\nu]) = \Pi(-\mathscr{F}_{\mu\nu}^\lambda Z_\lambda) = 0$$

Hence $\mathscr{F}_{\mu\nu}^\lambda = 0$, where $h[Z_\mu, Z_\nu]$ denotes the horizontal component of $[Z_\mu, Z_\nu]$. Thus, expressions (9) may be rewritten as

$$\begin{aligned} &[X_{ab}, X_{cd}] = \tfrac{1}{2} f_{ab,cd}^{ef} X_{ef} \\ &[Z_\mu, Z_\nu] = \tfrac{1}{2}\mathscr{F}_{\mu\nu}^{ab} X_{ab} \\ &[Z_\mu, X_{ab}] = 0 \end{aligned} \tag{9'}$$

Since

$$\begin{aligned} [X_{ab}, Z_\mu] &= [X_{ab}, \partial_\mu - \tfrac{1}{2}\mathscr{B}_\mu^{cd} X_{cd}] \\ &= -\tfrac{1}{2}(\partial_{ab}\mathscr{B}_\mu^{cd}) X_{cd} - \tfrac{1}{4}\mathscr{B}_\mu^{cd} f_{ab,cd}^{ef} X_{ef} \\ &= 0 \end{aligned}$$

then

$$\partial_{ab}\mathscr{B}^{cd}_{\mu} = -\tfrac{1}{2} f^{cd}_{ab,ef}\mathscr{B}^{ef}_{\mu}$$

Here $\partial_{ab} \equiv \partial x_{ab}$ is the directional derivative. It is known that $\mathscr{B}^{ab}_{\mu} = \mathscr{B}^{ab}_{\mu}(x, A)$, $A \in dS_{\lambda}(5)$, i.e., the connection not only relates to space-time points, but also depends on the elements of the gauge group.

On the bundle space $dSL(M_{\lambda(x)})$, by using the Jacobi identity

$$[X_{ab}, [Z_{\mu}, Z_{\nu}]] = [Z_{\mu}, [Z_{\nu}, X_{ab}]] + [Z_{\nu}, [X_{ab}, Z_{\mu}]] = 0$$

and expressions (10), we have

$$\begin{aligned}[X_{ab}, [Z_{\mu}, Z_{\nu}]] &= [Z_{\mu}, [Z_{\nu}, X_{ab}]] + [Z_{\nu}, [X_{ab}, Z_{\mu}]] \\ &= 0 \\ &= -\tfrac{1}{2}(\partial_{ab}\mathscr{F}^{cd}_{\mu\nu})X_{cd} - \tfrac{1}{4}\mathscr{F}^{cd}_{\mu\nu} f^{ef}_{ab,cd} X_{ef} \\ &= 0\end{aligned}$$

and so we have $\partial_{ab}\mathscr{F}^{ef}_{\mu\nu} = -\tfrac{1}{2} f^{ef}_{ab,cd}\mathscr{F}^{cd}_{\mu\nu}$; then $\mathscr{F}^{ab}_{\mu\nu} = \mathscr{F}^{ab}_{\mu\nu}(X, A)$, $A \in dS_{\lambda}(5)$, i.e., the curvature is a function of the coordinates of the space-time points and the fiber bundle coordinates. Since

$$\begin{aligned}[Z_{\mu}, Z_{\nu}] &= [\partial_{\mu} - \tfrac{1}{2}\mathscr{B}^{ab}_{\mu} X_{ab}, \partial_{\nu} - \tfrac{1}{2}\mathscr{B}^{cd}_{\nu} X_{cd}] \\ &= \tfrac{1}{2}\partial_{\mu}\mathscr{B}^{ab}_{\nu} X_{ab} + \tfrac{1}{2}\partial_{\nu}\mathscr{B}^{ab}_{\mu} X_{ab} - \tfrac{1}{2} f^{ab}_{cd,ef}\mathscr{B}^{cd}_{\mu}\mathscr{B}^{ef}_{\nu} X_{ab} \\ &= \tfrac{1}{2}\mathscr{F}^{ab}_{\mu\nu} X_{ab}\end{aligned}$$

so

$$\begin{aligned}\mathscr{F}^{ab}_{\mu\nu} &= \partial_{\mu}\mathscr{B}^{ab}_{\nu} - \partial_{\nu}\mathscr{B}^{ab}_{\mu} + \tfrac{1}{4} f^{ab}_{cd,ef}\mathscr{B}^{cd}_{\mu}\mathscr{B}^{ef}_{\nu} \\ &= \partial_{\mu}\mathscr{B}^{ab}_{\nu} - \partial_{\nu}\mathscr{B}^{ab}_{\mu} + \mathscr{B}^{a}_{\mu c}\mathscr{B}^{cd}_{\nu} - \mathscr{B}^{b}_{\mu c}\mathscr{B}^{ca}_{\nu} \qquad (11)\end{aligned}$$

The curvature $\mathscr{F}^{ab}_{\mu\nu}$ corresponds to the strength of the Yang-Mills field. By making use of the Jacobi identity

$$[Z_{\mu}, [Z_{\nu}, Z_{\lambda}]] + [Z_{\nu}, [Z_{\lambda}, Z_{\mu}]] + [Z_{\lambda}, [Z_{\mu}, Z_{\nu}]] = 0$$

we obtain the identity of gauge field strength:

$$\nabla_{\mu}\mathscr{F}^{ab}_{\nu\lambda} + \nabla_{\nu}\mathscr{F}^{ab}_{\lambda\mu} + \nabla_{\lambda}\mathscr{F}^{ab}_{\mu\nu} = 0$$

Here ∇_{μ} is the gauge-covariant derivative:

$$\nabla_{\lambda}\mathscr{F}^{ab}_{\mu\nu} = \partial_{\lambda}\mathscr{F}^{ab}_{\mu\nu} + \tfrac{1}{4} f^{ab}_{cd,ef}\mathscr{B}^{cd}_{\lambda}\mathscr{F}^{ef}_{\mu\nu}$$

By making use of the Jacobi identity

$$[X_{ab}, [X_{cd}, X_{ef}]] + [X_{cd}, [X_{ef}, X_{ab}]] + [X_{ef}, [X_{ab}, X_{cd}]] = 0$$

we obtain the Jacobi identity of the structure constants of the group $dS_\lambda(5)$:

$$f^{cc'}_{aa',bb'}f^{dd'}_{ee',cc'}+f^{cc'}_{bb',ee'}f^{dd'}_{aa',cc'}+f^{cc'}_{ee',aa'}f^{dd'}_{bb',cc'}=0$$

Thus, from the expression (11) we may write the dSL curvature as

$$\mathscr{F}_{\mu\nu}=(\mathscr{F}^a_{\mu\nu b})=\begin{pmatrix} F^i_{\mu\nu j}+I\mathring{V}^i_{\mu\nu j} & \mathring{T}^i_{\mu\nu} \\ I\mathring{T}_{\mu\nu j} & 0 \end{pmatrix}\in ds_\lambda(5)$$

Here

$$F^i_{\mu\nu j}=\partial_\mu B^i_{\nu j}-\partial_\nu B^i_{\mu j}+B^i_{\mu k}B^k_{\nu j}-B^k_{\nu j}B^j_{\mu k}\in ds(3,1)$$

is the Lorentz connection,

$$\mathring{T}^i_{\mu\nu}=\mathring{V}^i_{\nu,\mu}-\mathring{V}_{\mu,\nu}+(B^i_{\mu\nu}-B^i_{\nu\mu})|\lambda|^{1/2}$$
$$=|\lambda|^{1/2}T^i_{\mu\nu}+(|\lambda|^{1/2}_{,\mu}V^i_\nu-|\lambda|^{1/2}_{,\nu}V^i_\mu)$$

is the dSL torsion, and

$$\mathring{V}^i_{\mu\nu j}=\mathring{V}^i_\mu\mathring{V}_{\nu j}-\mathring{V}^i_\nu\mathring{V}_{\mu j}=|\lambda|\,V^i_{\mu\nu j}$$

In the above expressions

$$T^i_{\mu\nu}=V_{\nu,\mu}-V_{\mu,\nu}+B^i_{\mu\nu}-B^i_{\nu\mu} \tag{12}$$

is the Lorentz torsion and $\mathring{V}^i_\mu=|\lambda|^{1/2}V^i_\mu$.

3. DE SITTER–POINCARÉ GRAVITY DESCRIBED BY FIBER BUNDLE

In dSL gravity the frame vector e'_5 is at a particular position, such that $\forall x\in M_{\lambda(x)}$, $\exists M'_x$ and $E^5_{\lambda|x}$ so that

$$E^5_{\lambda|x}=M'_{\lambda|x}\otimes N_x$$

where $N_x=\{X_x\in E^5_{\lambda|x}|\langle X_x, Y_x\rangle=0$, for all $Y_X\in M'\}$, e'_5 is a basis of the one-dimensional normal space of $M_{\lambda(X)}$, and $\forall x\in M_{\lambda(x)}$, $\exists S^4_{\lambda(x)}$, the projection center $(O_{\lambda(x)})$ of $S^4_{\lambda(x)}$ is on N_x. The tangent point of the sphere $S^4_{\lambda(x)}$ and $M_{\lambda(x)}$ is denoted as Q_x. Then the radius $\overline{O_{\lambda(x)}Q_x}=1|\lambda|^{1/2}$ of the dS sphere $S^4_{\lambda(X)}$ reflects the localization of the umbilical point supersurface $M_{\lambda(x)}$, but this is not considered in the dSL frame, and thus the dSL frame is an insufficiently localized frame.

Now let us define the "full-localized" dSP moving frame

$$\{\overset{*}{e}_a\}=\{e'_i,\overset{*}{e}_5\}=\left\{e'_i,\frac{\alpha}{|\lambda|^{1/2}}e'_5\right\} \tag{13}$$

The frame coefficients $\{\overset{*}{W}{}^a_\mu\}$, $\{\overset{*}{W}{}^a_5\}$ are given by $\{\overset{*}{W}{}^a_\mu\}=\{V^i_\mu,0\}$, $\{\overset{*}{W}{}^a_5\}=\{0,\alpha/|\lambda|^{1/2}\}$. Here α is a dimensional constant, and $[\alpha]=L^{-1}$. For the dSP frame (13) we denote the gauge group by $\widehat{dS}_\lambda(5)$. Under the action of $\widehat{dS}_\lambda(5)$, the frame (13) is transformed as

$$\hat{e}_a = \hat{A}_a^b \overset{*}{e}_b, \qquad \hat{A}_a^b \in \widehat{dS}_\lambda(5)$$

If we require $\hat{e}_i = \hat{A}_i^b \overset{*}{e}_b = A_i^b e'_b$, $\hat{e}_5 = \hat{A}_5^a \overset{*}{e}_a = (\alpha/|\lambda|^{1/2}) e'_5$, we obtain

$$\hat{A}_i^j = A_i^j, \qquad \hat{A}_i^5 = \frac{|\lambda|^{1/2}}{\alpha} A_i^5, \qquad \hat{A}_5^5 = A_5^5, \qquad \hat{A}_5^i = \frac{\alpha}{|\lambda|^{1/2}} A_5^i$$

Thus, replacing the frame (1) by the dSP frame (13) and the group $dS_\lambda(5)$ by $\widehat{dS}_\lambda(5)$, and repeating the above procedure for building the dSL frame bundle, we may build a dSP frame bundle $dSP(M_{\lambda(X)}, \widehat{dS}_\lambda(5))$.

By use of the dSP frame we can realize the contraction from the dS bundle to the Poincaré bundle. The commutation relations of generators of the group $dS_\lambda(5)$ are

$$[X_{ab}, X_{cd}] = \eta_{ad} X_{bc} + \eta_{bc} X_{ad} - \eta_{ac} X_{bd} - \eta_{bd} X_{ac}$$

but for the group $\widehat{dS}_\lambda(5)$, we choose the generators as

$$X_{ij} = \xi_i \partial_j - \xi_j \partial_i \qquad \text{and} \qquad P_i = \frac{|\lambda|^{1/2}}{\alpha} X_5 = \frac{|\lambda|^{1/2}}{\alpha} (\xi_5 \partial_i - \xi_i \partial_5)$$

Their commutation relations are

$$\begin{aligned} &[X_{ij}, X_{kl}] = \eta_{il} X_{jk} + \eta_{jk} X_{il} - \eta_{ik} X_{jl} - \eta_{jl} X_{ik} \\ &[X_{ij}, P_k] = \eta_{jk} P_i - \eta_{ik} P_j \\ &[P_i, P_j] = -\frac{\lambda}{\alpha^2} X_{ij} \end{aligned} \tag{14}$$

As $\lambda \to 0$, in the neighborhood of the pole point one has

$$\lim_{\lambda \to 0} \frac{|\lambda|^{1/2}}{\alpha} (\xi_5 \partial_i - \xi_i \partial_5) = \frac{1}{\alpha} \partial_i$$

Assuming $\alpha = 1$, the expressions (14) become

$$\begin{aligned} &[X_{ij}, X_{kl}] \Rightarrow so(3, 1) \\ &[X_{ij}, \partial_k] = \eta_{jk} \partial_i - \eta_{ik} \nabla_j \end{aligned}$$

i.e., the Lie algebra $\widehat{ds}_\lambda(5)$ is contracted as the Lie algebra $iso(3, 1)$. Hence, the group $\widehat{dS}_\lambda(5)$ is contracted as $ISO(3, 1)$.

Below we find the elements of $ISO(3, 1)$ from the elements of the group $dS_\lambda(5)$, and obtain the degeneracy of the frame. Under the dSP frame and in the neighborhood of the pole point, we let the local dS sphere coordinates be $\{\overset{*}{\xi}{}^a\} = \{\xi^i, \overset{*}{\xi}{}^5\}$; the action of the projective change group $dS_\lambda(5)$ is given by

$$\begin{aligned}\beta\xi^i &= \hat{A}^i_a \overset{*}{\xi}{}^a \\ \beta\xi^5 &= \hat{A}^5_a \overset{*}{\xi}{}^a\end{aligned} \tag{15}$$

Here β is a nonzero factor. Since the group $ISO(3, 1)$ is a projective change group keeping the infinite surface invariant, then when $\overset{*}{\xi}{}^5 = 0$, we have $\xi^5 = 0$. Thus, from the second expression of (15) we obtain $\hat{A}^5_5 = 0$, and expressions (15) become

$$\begin{aligned}\beta\xi^i &= \hat{A}^i_a \overset{*}{\xi}{}^a \\ \beta\xi^5 &= \hat{A}^5_5 \overset{*}{\xi}{}^5\end{aligned} \tag{16}$$

Because $|\hat{A}| \neq 0$, $\hat{A}^5_i = 0$, so $\hat{A}^5_5 \neq 0$; dividing the rhs and lhs of the first expression in (15) by $\beta\xi^5$ and $\hat{A}^5_5\overset{*}{\xi}{}^5$, respectively, we obtain the transformation

$$\hat{y}^i = a^i_j \overset{*}{y}{}^j + \hat{a}^i_5$$

Here $\overset{*}{y}{}^i = \overset{*}{\xi}/\overset{*}{\xi}{}^5$ and $\hat{y}^i = \xi^i/\xi^5$ are both four-dimensional projective inhomogeneous coordinates, and

$$a^i_j = \frac{\hat{A}^i_j}{\hat{A}^5_5} = \frac{A^i_j}{A^5_5}, \qquad \hat{a}^i_5 = \frac{\hat{A}^i_5}{\hat{A}^5_5} = \frac{A^i_5}{A^5_5}\frac{1}{|\lambda|^{1/2}}, \qquad a^i_j \in SO(3, 1)$$

Since $\lambda \to 0$, $\hat{a}^i_5 \to \infty$ $(\hat{a}^5_i \to 0)$, then under the actions of the contracted group $ISO(3, 1)$, the transformation formula of the dSL frame becomes

$$\hat{e}_i = a^j_i e'_j + a^5_i \overset{*}{e}_5 = a^j_i e'_j, \qquad a^j_i a^5_i \in ISO(3, 1) \tag{17}$$

Here $a^5_i = 0$, $\{e'_i\}$ is the Lorentz frame at $O_{\lambda(x)}$, and $\{\hat{e}_i\}$ is another Lorentz frame under $ISO(3, 1)$. At the same time, for the fifth frame vector we have

$$\hat{e}_5 = \hat{a}^i_5 e'_i + a^5_5 \overset{*}{e}_5 = \hat{a}^i_5 e'_5 + \overset{*}{e}_5, \qquad a^5_5(\in ISO(3, 1)) = 1$$

The above expression may be written in the form

$$e_5 = \frac{A^i_5}{A^5_5} e'_i + e'_5 = a^i_5 e'_i + e'_5 \tag{18}$$

Here $a^i_5(\in ISO(3, 1)) = A^i_5/A^5_5$. Combining expression (17) with (18), we may write the group $ISO(3, 1)$ as the form

$$\begin{pmatrix} a^i_j & a^i_5 \\ 0 & 1 \end{pmatrix}, \qquad a^i_j \in SO(3, 1), \quad a^i_5 \text{ a real number}$$

This form is just the matrix fashion of $ISO(3, 1)$ and has been used to build the Poincaré affine frame bundle (Shao Changgui and Xu Banqing, 1986).

From expressions (17) and (18), we know that as the gauge group is contracted, the dSP frame degenerates into the form $\{\hat{e}_i, e_5\}$, where, since e_5 denotes the translation of the origin of the frame $\{\hat{e}_i\}$ in the space M'_x under the action of $ISO(3,1)$, the frame $\{\hat{e}_i, e_5\}$ is a Poincaré moving frame.

Since $\lambda \to 0$, we have $M_{\lambda(x)} \to M$; then $\forall x \in M$, we can obtain a series of frames $\{\hat{e}_i, e_5\}_x$, which may be considered as a fiber over point x. Taking the union $P(M) = \bigcup_{x\in M} \{\hat{e}_i, e_5\}_x$ for all $x \in M$, we obtain a Poincaré affine frame bundle $P(M) = P(M, ISO(3,1))$.

Since, when $\lambda \to 0$ we have $\widehat{dS}_\lambda(5) \to ISO(3,1)$, $\widehat{ds}(5) \to iso(3,1)$, $M_{\lambda(X)} \to$ RC space-time, and

$$\{\hat{e}_a\} \to \{\hat{e}_i, e_5\} \tag{19}$$

then we have frame bundle $P(M_{\lambda(x)}, \widehat{dS}_\lambda(5)) \to$ frame bundle

$$P(M, ISO(3,1))$$

When the dS gauge group is restricted on its Lorentz subgroup, the structure group of the bundle $dSP(M_{\lambda(x)})$ and the bundle $dSL(M_{\lambda(x)})$ both take the form

$$\begin{pmatrix} SO(3,1) & 0 \\ 0 & 1 \end{pmatrix}$$

and the roles of the fifth vectors of frames on the two bundles are degenerate; these two bundles thus both degenerate into their Lorentz subbundle (Shao Changgui *et al.*, unpublished)

$$L(M_{\lambda(x)}) = P(M_{\lambda(x)}, dS_\lambda(5)|_{SO(3,1)})$$

Under the dSP frame (13) the Gauss and Weingarten formulas are

$$De'_i = B^j_{\mu_i}\, dx^\mu \otimes e'_i - I\overset{*}{\omega}_{ij}\theta'^j \otimes \overset{*}{e}_5$$
$$D\overset{*}{e}_5 = -\overset{*}{\omega}_{kj}\eta^{ik}\theta'^j \otimes e'_i$$

Here the second fundamental metric is

$$\overset{*}{\omega}_{ij} = -\alpha\eta_{ij} \tag{20}$$

Under the frame the three fundamental forms of surface theory satisfy the following relation:

$$\mathrm{I} = -\alpha\,\mathrm{II} = \alpha^2\,\mathrm{III}$$

Using expression (20), we may write the Gauss and Weingarten formulas as

$$\begin{aligned} De'_i &= B^j_{\mu_i}\, dx^\mu \otimes e'_j + I\theta'_i \otimes \overset{*}{e}_5 \\ D\overset{*}{e}_5 &= \alpha\theta'^i \otimes e'_i \end{aligned} \tag{21}$$

But under the dSP frame the connection corresponding to expression (4) should be written as

$$De'_i = \overset{*}{\mathscr{B}}{}^j_{\mu_i}\, dx^\mu \otimes e'_j + \overset{*}{\mathscr{B}}{}^5_{\mu_i} \otimes \overset{*}{e}_5$$
$$De_5 = \mathscr{B}^i_{\mu 5}\, dx^\mu \otimes e'_i \tag{22}$$

Comparing expression (21) with (22), we obtain $\overset{*}{\Omega}{}^j_i = B^j_{\mu_i}\, dx^\mu$, $\overset{*}{\Omega}{}^i_5 = \alpha\theta^i$, and $\overset{*}{\Omega}{}^5_i = I\overset{*}{\Omega}_{i5}$. Then, under the cross section $\overset{*}{\sigma}(x) = \{\overset{*}{e}_a(x)\}$ on the bundle $dSP(M_{\lambda(X)})$, the dSP connection is

$$\overset{*}{\mathscr{B}}_\mu = (\overset{*}{\mathscr{B}}{}^b_{\mu a}) = \begin{pmatrix} B^j_{\mu_i} & \alpha V^j_\mu \\ \alpha I V_{\mu i} & 0 \end{pmatrix} \in \widehat{ds}_\lambda(5)$$

For the dSP bundle, with the exception of the connection, we may build the same theory as the dSL bundle. Here we only give the dSP curvature:

$$\overset{*}{\mathscr{F}}_{\mu\nu} = (\overset{*}{\mathscr{F}}{}^b_{\mu\nu a}) = \begin{pmatrix} F^j_{\mu\nu i} + \alpha^2 I V^j_{\mu\nu i} & \alpha T^j_{\mu\nu} \\ \alpha I T_{\mu\nu_i} & 0 \end{pmatrix} \in \widehat{ds}_\lambda(5) \tag{23}$$

Here $F^j_{\mu\nu i}$ is the Lorentz curvature, $T^i_{\mu\nu}$ is the Lorentz torsion, and $V^i_{\mu\nu j} = V^i_\mu V_{\nu j} - V^i_\nu V_{\mu j}$.

As $\lambda \to 0$, the dSP bundle is contracted to the principal bundle $P(M)$, the algebra $\widehat{ds}_\lambda(5)$ is contracted to the Lie algebra $iso(3, 1)$, and then the connection of $P(M)$ will take its values on the Lie algebra $iso(3, 1)$. Thus, the curvature will also degenerate. Hence, as $\lambda \to 0$ we also have

$$\text{dSP connection} \begin{pmatrix} B^j_{\mu_i} & \alpha V^j_\mu \\ I\alpha V_{\mu_i} & 0 \end{pmatrix} \to \text{Poincaré connection } (B^j_{\mu_i}, V^j_\mu)$$

$$\text{dSP curvature} \begin{pmatrix} F^j_{\mu\nu i} + \alpha^2 I V^j_{\mu\nu i} & \alpha T^j_{\mu\nu} \\ \alpha I T_{\mu\nu_i} & 0 \end{pmatrix} \to \text{Poincaré curvature } (F^j_{\mu\nu i}, T^j_{\mu\nu})$$

Now we see that taking the connection of the principal bundle $P(M)$ as the different components, we can construct the dSP connection, but the latter is a 5×5 matrix; thus, the dSP connection and curvature are different from those of the former. The connection and the curvature of $P(M)$ do not have the 5×5 matrix form; then the gravitational gauge theory obtained from the dSP bundle is not the same as that obtained from the Poincaré bundle, unless the former degenerates into the latter. This difference is also reflected in the construction of the field equations.

The dSL curvature and dSP curvature discussed in this paper are two types of different curvatures; we may obtain at least two types of dS gravitational gauge theories (Cho, 1975; Chang *et al.*, 1976).

4. FIELD EQUATIONS OF DE SITTER–POINCARÉ GRAVITY

The Lagrangian used in dSP gravity is chosen as

$$\mathcal{L} = \mathcal{L}_g + \frac{1}{2\varepsilon}\mathcal{L}_m V$$

$$= -\tfrac{1}{4} V \operatorname{Tr}(\overset{*}{\mathcal{F}}_{\mu\nu}\overset{*}{\mathcal{F}}^{\mu\nu}) + \frac{1}{2\varepsilon} V\mathcal{L}_m(\psi, \psi_{\|\mu}, g_{\mu\nu})$$

Here

$$\mathcal{L}_g = -\tfrac{1}{4} V \operatorname{Tr}(\overset{*}{\mathcal{F}}_{\mu\nu}\overset{*}{\mathcal{F}}^{\mu\nu}) = -\tfrac{1}{8} V \overset{*}{\mathcal{F}}{}^{a}_{\mu\nu b}\overset{*}{\mathcal{F}}{}^{\mu\nu b}_{a}$$

is the Lagrangian of the gravitational gauge field, $\mathcal{L}_m = \mathcal{L}_m(\psi, \psi_{\|\mu}, g_{\mu\nu})$ is the Lagrangian of the matter field, the symbol $\|$ is the twofold covariant derivative with respect to the natural frame and the Lorentz moving frame, ε is the coupling constant of dSP gravity, and $V = \det|V^i_\mu| = (-g)^{1/2}$. In $\mathcal{L}_m$ the ordinary partial derivative $\psi_{,\mu}$ used in ordinary field theory should be replaced by the above twofold covariant derivative, and the metric η_{ij} by $g_{\mu\nu}$. Now we take the variation with respect to $\mathcal{B}^{\ b}_{\mu a}$ and use the variational principle; then the Euler equations are

$$\frac{1}{2\varepsilon}\frac{\partial(\mathcal{L}_m V)}{\partial \overset{*}{\mathcal{B}}{}^{ab}_{\mu}} - \frac{1}{2\varepsilon}\partial_\nu\left(\frac{\partial(\mathcal{L}_m V)}{\partial \overset{*}{\mathcal{B}}{}^{ab}_{\mu,\nu}}\right) + \frac{\partial(\mathcal{L}_g V)}{\partial \overset{*}{\mathcal{B}}{}^{ab}_{\mu}} - \partial_\nu\left(\frac{\partial(\mathcal{L}_g V)}{\partial \overset{*}{\mathcal{B}}{}^{ab}_{\mu,\nu}}\right) = 0$$

Applying expression (23), we obtain

$$\overset{*}{\mathcal{F}}{}^{\mu\nu}_{ab/\!/\nu} = \frac{-1}{\varepsilon} S^\mu_{ab} - K^\mu_{ab} \tag{24}$$

Here

$$S^\mu_{ab} = \frac{1}{V}\frac{\delta(V\mathcal{L}_m)}{\delta \overset{*}{\mathcal{B}}{}^{ab}_{\mu}}$$

is the spin current of the matter system, the symbol $/\!/$ is the twofold covariant derivative with respect to the natural frame and the dSP frame, and

$$(K^{\mu a}_b) = \frac{-1}{\alpha^2 I}\mathcal{T}^{\lambda\mu}\begin{pmatrix} 0 & \alpha V^i_\lambda \\ \alpha I V & 0 \end{pmatrix}$$

where

$$\mathcal{T}^{\mu\nu} = \tfrac{1}{4} g^{\mu\nu}(\overset{*}{\mathcal{F}}{}^{j}_{\sigma\lambda i}\overset{*}{\mathcal{F}}{}^{\sigma\lambda i}_{j}) - (\overset{*}{\mathcal{F}}{}^{\nu j}_{\sigma_i}\overset{*}{\mathcal{F}}{}^{\mu\sigma i}_{j})$$

From (1) we also find

$$\overset{*}{\mathcal{F}}{}^{\mu\nu 5}_{i/\!/\nu} = \alpha I T^{\mu\nu}_{i\|\nu} + \alpha I(F^\mu_i - 3\alpha^2 I V^\mu_i)$$

In order to find the relation between the dSP gravity and general relativity, we rewrite equations (24) as the following two sets:

$$\overset{*}{\mathscr{F}}{}^{\mu\nu 5}_{i/\!/\nu} = \frac{-2}{\alpha\varepsilon} T^{\mu}_{i} + \frac{1}{\alpha}\mathscr{T}^{\mu}_{i}$$
$$\overset{*}{\mathscr{F}}{}^{\mu\nu}_{ij/\!/\nu} = \frac{-1}{\varepsilon} S^{\mu}_{ij} \tag{25}$$

Here

$$T^{\mu}_{i} = \frac{1}{2}\frac{\delta(\mathscr{L}_m V)}{V\delta V^{i}_{\mu}}$$

is the mass tensor of matter. When we rewrite the covariant derivative $/\!/$ used in the above two sets of equations as the twofold covariant derivative with respect to natural and Lorentz frames, the gauge field equations (25) become

$$\tfrac{1}{2}T^{\mu\nu}_{i\|\nu} + F^{\mu}_{i} - \tfrac{1}{2}FV^{\mu}_{i} + \tfrac{3}{2}F^{\mu}_{i} - \frac{3}{2}\frac{\alpha^2}{I}V^{\mu}_{i}$$
$$= -\frac{1}{\alpha^2 I\varepsilon}T^{\mu}_{i} + \frac{1}{2\alpha^2 I}t^{\mu\nu}V_{\nu i} + \tau^{\mu\nu}V_{\nu i} \tag{26}$$

$$F^{\mu\nu}_{ij\|\nu} - \varepsilon V_{\nu[j}T^{\mu\nu}_{i]} = -\varepsilon S^{\mu}_{ij} \tag{27}$$

where [ ] is the antisymmetric symbol and

$$t^{\mu\nu} = \tfrac{1}{4}g^{\mu\nu}F^{i}_{\lambda\sigma j}F^{\lambda\sigma j}_{i} - F^{\mu\lambda j}_{i}F^{\nu i}_{\lambda j}$$
$$\tau^{\mu\nu} = \tfrac{1}{4}g^{\mu\nu}\,\mathrm{Tr}(t_{\lambda\sigma}t^{\lambda\sigma}) - \mathrm{Tr}(t^{\mu\lambda}t^{\nu}_{\lambda})$$

Let

$$\frac{3}{2}\frac{\alpha^2}{I} = \Lambda, \qquad \frac{1}{\alpha^2 I\varepsilon} = C$$

Here Λ is the cosmological constant, and C is the Einstein gravitational constant. Equations (26), (27) may be written as

$$-\tfrac{1}{2}T^{\mu\nu}_{i\|\nu} + G^{\mu}_{i} - \Lambda V^{\mu}_{i} + \tfrac{3}{2}F^{\mu}_{i} = -CT^{\mu}_{i} + \tfrac{1}{2}C\varepsilon t^{\mu}_{i} + \tau^{\mu}_{i} \tag{28}$$

$$F^{\mu\nu}_{ij\|\nu} - \varepsilon V_{\nu[j}T^{\mu\nu}_{i]} = -\varepsilon S^{\mu}_{ij} \tag{29}$$

Here $G^{\mu}_{i} = F^{\mu}_{i} - \tfrac{1}{2}FV^{\mu}_{i}$ is the Einstein tensor.

Comparing the gravitational gauge field equations (28) and (29) with the gravitational gauge field equations of Poincaré gravity (Shao Changgui and Xu Banqing, 1986).

$$G^{\mu}_{i} = -CT^{\mu}_{i} + \rho t^{\mu}_{i} + \rho'\tau^{\mu}_{i}$$
$$-\rho F^{\mu\nu}_{ij\|\nu} = CS^{\mu}_{ij} + H^{\mu}_{ij} - \rho' T^{\mu}_{ij} \tag{30}$$

we see that the differences are that for the dSP gravity there are terms of the covariant divergence $-\frac{1}{2}T^{\mu\nu}_{i\|\nu}$ and the curvature tensor $\frac{3}{2}F^{\mu}_{i}$, two terms not in the gravitational gauge field equations of Poincaré gravity. The appearance of the covariant divergence term is based on the matrix fashion of the curvature of dSP gravity. The appearance of an additional curvature tensor term is based on the hypothesis that the dS universe has constant curvature. In equations (29) one does not find the same cotorsion term H^{μ}_{ij} as in equations (30), which is also based on the dS global invariance of the dS universe. The Poincaré gravity is based on the supposition that the space-time is obtained by localizing the flat Minkowski space-time, but the de Sitter gravity is based on the supposition that space-time is obtained by localizing the dS sphere holding constant curvature. Poincaré gravity is a version of a limiting degeneration of dSP gravity; it is not a subgravity of the latter. The appearance of the additional curvature term $\frac{3}{2}F^{\mu}_{i}$ just reflects this effect.

For the torsion-free case the dSP gravitational gauge field equations become

$$\begin{gathered} G^{\mu}_{i}-\Lambda V^{\mu}_{i}+\tfrac{3}{2}F^{\mu}_{i}=-CT^{\mu}_{i}+\tfrac{1}{2}C\varepsilon t^{\mu}_{i} \\ F^{\mu\nu}_{ij\|\nu}=-\varepsilon S^{\mu}_{ij} \end{gathered} \tag{31}$$

Equations (31) indicate that the dSP gravitational gauge field equations discussed in this article do not degenerate into extended Einstein–Yang equations, except that the dS effect discussed above degenerates.

5. FIELD EQUATIONS OF DE SITTER–LORENTZ GRAVITY

In finding the field equations, we first suppose that there is no relation between the potential $\mathscr{B}'^{b}_{\mu a}$ and the first fundamental metric of the space-time manifold. Thus, using a similar method as in dSP gravity to find the field equations, we may consider that the V^{i}_{μ} contained in the Lagrangian

$$\mathscr{L}_{g}=-\tfrac{1}{8}V\mathscr{F}^{a}_{\mu\nu b}\mathscr{F}^{\mu\nu b}_{a}$$

is independent of the fifth component of the dSL gravitational potential. Thus, we find the field equations

$$\mathscr{F}^{\mu\nu}_{ab/\!/\nu}=-\frac{1}{\varepsilon}S^{\mu}_{ab}$$

and we can write the above equations as two sets

$$I\mathring{T}^{\mu\nu}_{i\|\nu}+I(F^{\mu}_{i}-3I\mathring{V}^{\mu}_{i})=-\frac{2}{\varepsilon}V^{\mu}_{i} \tag{32}$$

$$F^{\mu\nu}_{ij\|\nu}-\varepsilon\mathring{V}_{\nu[j}\mathring{T}^{\mu\nu}_{i]}=-\varepsilon S^{\mu}_{ij} \tag{33}$$

These differ from equations (28) and (29) in that equations (32) and (33) contain terms with factors $|\lambda|^{1/2}$ and $|\lambda|^{1/2}_{,\mu}$, which are relevant to the localization of space-time, and thus may be used to discuss the influence of space-time localization on the field equations.

Now suppose that the space-time localization is trivial, i.e., $|\lambda|^{1/2}_{,\mu}=0$; the dSL frame bundle will degenerate into a trivial bundle, and suppose space-time is torsion free; thus, equations (32), (33) become

$$F_i^\mu - 3I|\lambda|^{1/2} V_i^\mu = -\frac{2}{\varepsilon I} T_i^\mu \tag{34}$$

$$F_{ij\|\nu}^{\mu\nu} = -\varepsilon S_{ij}^\mu \tag{35}$$

Here let

$$-3I|\lambda|^{1/2} = \Lambda, \qquad \frac{2}{\varepsilon I} = C \tag{36}$$

Then equations (34), (35) may be written as

$$F_i^\mu + \Lambda V_i^\mu = -CT_i^\mu \tag{37}$$

$$F_{ij\|\nu}^{\mu\nu} = -\frac{2}{CI} S_{ij}^\mu \tag{38}$$

In expression (36) the appearance of I may make it possible to choose between gauge groups $dS(3,2)$ and $dS(4,1)$.

When the matter system has no spin, equations (37), (38) become

$$F_i^\mu + \Lambda V_i^\mu = -CT_i^\mu$$

$$F_{ij\|\nu}^{\mu\nu} = 0$$

or

$$R_\nu^\mu + \Lambda g_\nu^\mu = -CT_\nu^\mu \tag{39}$$

$$R_{\nu\lambda\sigma}^{\mu\|\nu} = 0 \tag{40}$$

where R_ν^μ is the Ricci curvature tensor, and $R_{\nu\lambda\sigma}^\mu$ is the Riemann curvature tensor. Equations (39) and (40) correspond to the Einstein equations and the Yang equations, respectively.

REFERENCES

Chang, L. N., Macrae, K. I., and Massouri, F. (1976). *Physical Review D*, **13**, 235.

Cho, Y. M. (1975). *Journal of Mathematical Physics*, **16**, 2029.

Kobayachi, S., and Nomizu, K. (1963). *Foundations of Differential Geometry*, Vol. 1, Interscience, New York.

Massouri, F., and Chang, L. N. (1976). *Physical Review D*, **13**, 3192.

Shao Changgui, and Xu Banqing (1986). *International Journal of Theoretical Physics*, **4**, 347.

Shao Changgui, Guo Youzhong, and Xu Bangqing, *SO*(3, 1) gauge theory of gravity, unpublished.